

# Insights from Curvy RED (Random Early Detection)

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## Abstract

Active queue management (AQM) drops packets early in the growth of a queue, to prevent a capacity-seeking sender (e.g. TCP) from keeping the buffer full. An AQM can mark instead of dropping packets if they indicate support for explicit congestion notification (ECN [RFB01]). Two modern AQMs (PIE [PNB<sup>+</sup>15] and CoDel [NJ12]) are designed to keep queuing delay to a target by dropping packets as load varies.

This memo uses Curvy RED and an idealised but sufficient model of TCP traffic to explain why attempting to keep delay constant is a bad idea, because it requires excessively high drop at high loads. This high drop itself takes over from queuing delay as the dominant cause of delay, particularly for short flows. At high load, a link is better able to preserve reasonable performance if the delay target is softened into a curve rather than a hard cap.

The analysis proves that the same AQM can be deployed in different parts of a network whatever the capacity with the same optimal configuration.

A surprising corollary of this analysis concerns cases with a highly aggregated number of flows through a bottleneck. Although aggregation reduces queue variation, if the target queuing delay of the AQM at that bottleneck is reduced to take advantage of this aggregation, TCP will still increase the loss level because of the reduction in round trip time. The way to resolve this dilemma is to overprovision (a formula is provided).

Nonetheless, for traffic with ECN enabled, there is no harm in an AQM holding queuing delay constant or configuring an AQM to take advantage of any reduced delay due to aggregation without over-provisioning. Recently, the requirement of the ECN standard [RFB01] that ECN must be treated the same as drop has been questioned. The insight that the goals of an AQM for drop and for ECN should be different proves that this doubt is justified.

## 1 Curvy RED

Curvy RED [DSBTB15] is an active queue management (AQM) algorithm that is both a simplification

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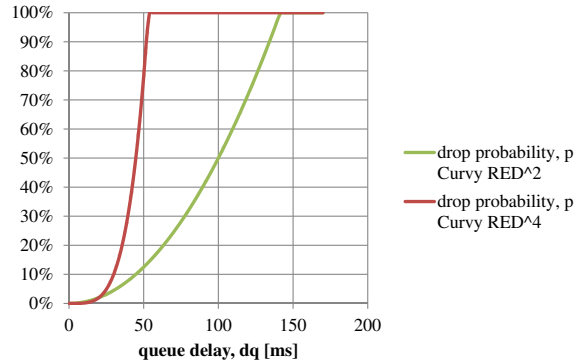


Figure 1: Two Example Curvy RED algorithms

and a generalisation of Random Early Detection (RED [FJ93]). Two examples are shown in Figure 1 and Figure 2 shows a close-up of their normal operating regions.

The drop probability,  $p$ , of a Curvy RED AQM is:

$$p = \left( \frac{d_q}{D_q} \right)^u, \quad (1)$$

where  $d_q$  is averaged queue delay. The two parameters are:

$u$ : the exponent (cUrviness) of the AQM;

$D_q$ : the slop, i.e.  $d_q$  where  $p$  hits 100%.

An implementation would not be expected to vary cUrviness,  $u$ . Rather an optimal value of  $u$  would be determined in advance.

The queuing delay used by Curvy RED is averaged, but averaging is outside the scope of his memo, which is only concerned with insights from steady-state conditions.

The slopes  $D_q$  of the curves in Figure 1 are arranged so that they both pass through (20 ms, 2%), which we call the design point.<sup>1</sup> All the curves in Figure 2 and Figure 3 are arranged to pass through this same

<sup>1</sup>The values are an arbitrary example, not a recommendation.

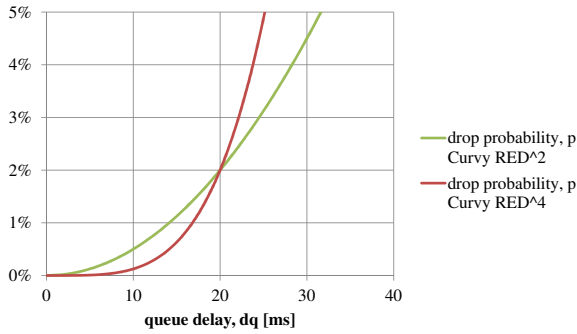


Figure 2: Usual Operating Region of The Same Two Example Curvy RED AQM algorithms

design point, which makes them comparable over the operating region of about 0–40 ms either side of this point, shown in [Figure 2](#).

In the following, the dependence of both drop and queuing delay on load will be derived. The approach is relatively simple. The rate of each flow has to reduce to fit more flows into the same capacity, the number of flows being a measure of increasing load. The formula for the rate of a TCP flow is well known; flow rate reduces if either drop or round trip time (RTT) increases, and RTT increases if queuing delay increases. The Curvy RED formula constrains the relationship between drop and queuing delay, so one can be substituted for the other in the TCP formula to cast it solely in terms of either drop or queuing delay, rather than both. Therefore either drop or queuing delay can be stated in terms of the number of flows sharing out the capacity.

[Figure 3](#) shows the resulting dependence of both queuing delay (left axis) and drop probability (right axis) on load. There is a pair of curves for each of a selection of different values of curviness,  $u$ , in the Curvy RED algorithm. The rest of this section gives the derivation of these curves, and defines the normalised load metric used for the horizontal axis.

In practice, traffic is not all TCP and TCP is not all Reno. Nonetheless, to gain insight it will be sufficient to assume all flows are TCP Reno. Focusing on Reno is justified for bandwidth delay products typical in today’s public Internet, where TCP Cubic typically operates in a TCP Reno emulation mode (e.g. with 20ms round trip time, Cubic remains in Reno mode up to 500Mb/s). The following Reno-based analysis would apply for Cubic in Reno mode, except with a slightly different constant of proportionality. Even if the dominant congestion control were Cubic in its pure Cubic mode, the exponents and constants would be different in

form, but the structure of the analysis would be the same.

In typical traffic a large proportion of TCP flows are short and complete while still in slow start. However, again for intuitive simplicity, only flows in congestion avoidance will be considered.<sup>2</sup>

The load on a link is proportional to the number of simultaneous flows being transmitted,  $n$ . If the capacity of the link (which may vary) is  $X$  and the mean bit-rate of the flows is  $x$ , then:

$$n = \frac{X}{x}. \quad (2)$$

The rate of a TCP flow depends on round trip delay,  $d_R$  and drop probability,  $p$ . A precise but complex formula for this dependence has been derived, but the simplest model [[MSMO97](#)] will suffice for insight purposes:

$$x = \frac{Ks}{d_R\sqrt{p}}, \quad (3)$$

where  $s$  is the maximum segment size (MSS) of TCP and  $K$  is  $\sqrt{3}/2$  for TCP Reno (or 1.68 for Cubic in Reno mode).

The round trip time,  $d_R$  consists of the base RTT  $D_R$  plus queuing delay  $d_q$ , such that:

$$d_R = D_R + d_q \quad (4)$$

For a set of flows with different base round trip times, it is sufficient to define  $D_R$  as the harmonic mean of the set of base RTTs.  $d_q$  is common to all the flows, because they all pass through the same queue<sup>3</sup>, so then  $d_R$  is the average RTT including this queuing delay  $d_q$ .

Substituting [Equation 3](#) [Equation 4](#) in [Equation 2](#):

$$n = \frac{X(D_R + d_q)}{Ks} \sqrt{p}. \quad (5)$$

By substituting from [Equation 1](#) into [Equation 5](#), the number of flows can be given as a function of either queuing delay  $d_q$  or loss probability,  $p$ :

$$n = \frac{X(D_R + d_q)}{Ks} \left( \frac{d_q}{D_q} \right)^{u/2} \quad (6)$$

$$n = \frac{X(D_R + D_q p^{1/u}) p^{1/2}}{Ks} \quad (7)$$

<sup>2</sup>If necessary, the traffic can be characterised as an *effective* number,  $n$ , of Reno flows in congestion avoidance, for instance by reverse engineering the output of an algorithm designed to measure queue variation  $\nu$  such as ADT [[SSK06](#)], i.e.  $n = (\text{BDP}/\nu)^2$ .

<sup>3</sup>Assuming the common case of a single bottleneck.

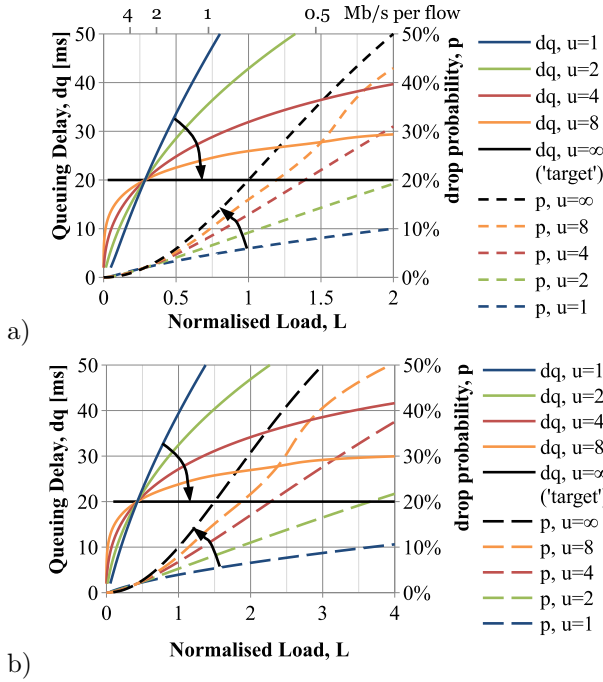


Figure 3: Dilemma between Two Impairments: Delay and Loss against Normalised Load; for TCP Reno and the Curvy RED Algorithm with Increasing Curviness,  $u$ ; a)  $D_R = 20$  ms; b)  $D_R = 10$  ms

Equation 6 & Equation 7 are plotted against normalised load in Figure 3 for an example set of Curvy RED algorithms with curviness parameters  $u = 1, 2, 4, 8, \infty$ , with base RTT  $D_R = 20$  ms.  $u = \infty$  represents an algorithm like PIE or CoDel that attempts to clamp queuing delay to a target value whatever the load.

How to read Figure 3: For any one value of  $u$ , only two plots apply. Each pair shares a common colour. The solid line in each pair shows growth in queuing delay (left axis) with load and the dashed line shows growth in drop (right axis). For instance, for  $u = 1$ , the two blue plots apply, which are uppermost and lowermost. As the value used for  $u$  increases, the relevant pair of plots to use progress inwards; down from the top and up from the bottom, like a closing pair of pincers.

Normalised load  $L$  is used as the horizontal axis, because it is proportional to the number of flows  $n$  but scales with link capacity  $X$ , by the following relationship:

$$L = \frac{Ks}{D_R} \frac{n}{X}, \quad (8)$$

where  $Ks/D_R$  can be considered constant, at least as long as MSS and the typical topology of the Internet are constant. The plot uses  $s = 1500$  B.

It can be seen that, as the curviness parameter is increased, the AQM pushes harder against growth

of queuing delay as load increases. However, given TCP is utilising the same capacity, it has to cause more loss (right axis) if it cannot cause more queuing delay (left axis).

In the extreme, infinite curviness represents the intent of AQMs such as CoDel and PIE that aim to clamp queuing delay to a target value (the black horizontal straight line). As a consequence, the TCP flows force loss to rise more quickly with load (the black dashed line). The experiments comparing Curvy RED with RED, PIE and fq\_CoDel in [DSBTB15] in a realistic broadband testbed verify this analysis, at least for  $u = 2$ . Indeed the high drop levels found with PIE and fq\_CoDel in these experiments motivated the analysis in this report.

To reduce losses at high load, the default target delay of fq\_CoDel probably needs to be increased to a value closer to PIE's target default of 20 ms. However then, when load is below the design point, both PIE and CoDel will tend to allow a small standing queue to build so that the queue is sufficient to reach the target delay. Whereas Curvy RED should give lower queuing delay when load is light.

Normalised load is plotted on the horizontal axis to make the plots independent of capacity as long as individual flows have the same rate. As an example, we will set the constants to average base RTT,  $D_R = 20$  ms and MSS,  $s = 1500$  B. Then, if the average rate of individual flows,  $x = 1$  Mb/s normalised load will be  $L = 1500 * 8 * \sqrt{3}/2 / (0.02 * 1E6) = 0.73$ , whether there are  $n = 4$  flows in  $X = 4$  Mb/s of capacity, or  $n = 400$  flows in  $X = 400$  Mb/s.

From Equation 8 it can be seen that normalised load depends on the choice of average base delay used to characterise all the paths,  $D_R$ . The horizontal scale of Figures 3a) and 3b) has been contrived so that a vertical line through them both represents the same mean rate per flow,  $x$ .<sup>4</sup> A few selected values of rate per flow are shown along the top of Figure 3a).

For instance, if  $D_R = 20$  ms then  $x$  is 1 Mb/s when  $L = 0.73$ , but if  $D_R = 10$  ms,  $x$  is 1 Mb/s when  $L = 1.46$ . At twice the normalised load in both plots,  $x$  will halved. As well as this scaling, there is still residual dependence on  $D_R$  in the shape of the plots as well, even though normalised load is defined to include average RTT  $D_R$ .

<sup>4</sup>Interestingly, normalised load can be expressed as proportionate to the product of number of flows and the ratio of two delays:  $L = nKD_s/D_R$ , where  $D_s$  is the serialisation delay of a maximum sized segment, because  $D_s = s/X$ .

## 2 Invariance with Capacity

### 2.1 Does Flow Aggregation Increase or Decrease the Queue?

There used to be a rule of thumb that a router buffer should be sized (in bytes) for the bandwidth delay product. Then, in 2004 Appenzeller et al. [AKM04, GM06] pointed out that for large aggregates  $\text{BDP}/\sqrt{n}$  would be sufficient. This applies only if TCP's sawteeth are desynchronized so that the variance of all the sawteeth shrinks with the square root of the number of flows.<sup>5</sup>

A link for  $100\times$  more flows will typically be sized with  $100\times$  more capacity, so the queue will drain  $100\times$  faster. Therefore, if the buffer is sized for  $\text{BDP}/\sqrt{n}$ , queuing delay will be  $\sqrt{n}\times$  ( $=10\times$ ) lower.<sup>6</sup>

In the previous section we showed that queuing delay *grows* with number of flows. How does this reconcile with the idea that buffers can *shrink* as more flows are aggregated?

It is certainly true that, with more flows, variability of the queue will shrink. So, if curvy RED allows the extra flows to increase the queue, most of the variation will be at the tail of the queue in front of a small standing queue at the head. However, without more capacity, the alternative would be for the AQM to introduce more drop to prevent the additional delay—the dilemma in Figure 3 cannot be escaped<sup>7</sup>.

Nonetheless, if the greater number of flows is expected to persist, capacity can be increased. This creates enough space again for each flow, so less queuing delay and less loss is needed. For example, if there are permanently ten times more flows, then provisioning ten times more capacity will bring *normalised* load back to its original value, because normalised load,  $L \propto n/X$ .

Note that our definition of normalised load also factors out a change in MSS. For a given capacity and number of flows, increasing the MSS increases the normalised load. It may be counter-intuitive that the same bit-rate in larger packets will change normalised load. This is an artefact of the design of the TCP Reno algorithm (and most other TCP variants), which congests the link more (more delay and/or drop) for a larger MSS, because it adds one MSS to the load per RTT (the additive increase).

<sup>5</sup>This square root comes from the Central Limit Theorem and has nothing to do with the TCP Reno rate equation.

<sup>6</sup>Note that this square-root law does not apply indefinitely; only for medium levels of aggregation. Once the queue size has reduced to about half a dozen packets, it does not get any smaller with increased aggregation [GM06].

<sup>7</sup>except using ECN —see §3

### 2.2 AQM Configuration and Scale

AQM configuration should be invariant with link capacity, but it will need to change in environments where the expected range of RTTs is significantly different. Therefore, AQM configurations in a data centre will be different, not because of high link capacities, but because of shorter RTTs.

The parameter  $D_q$  represents a delay well outside the normal operating region, so it has no intuitive meaning. Therefore it is better to configure the AQM against a design point, such as  $d_q = 20\text{ms}, p = 2\%$ . The operator will need to set this design point where delay and loss start to become troublesome for the most sensitive applications (e.g. interactive voice). By setting the AQM to pass through this point, it will ensure a good compromise between delay and loss. Then the most sensitive application will survive at the highest possible load, rather than holding one impairment unnecessarily low so that the other is pushed so high that it breaks the sensitive app.

The ideal curviness of the AQM depends on the dominant congestion control regime (this memo is written assuming TCP Reno is dominant, which includes Cubic in Reno mode). For a particular dominant congestion control, curviness depends only on the best compromise to resolve the dilemma in Figure 3, and can otherwise be assumed constant.

Having determined an AQM configuration as above, it will be applicable for all link rates as long as expected round trip times are unchanged. It would seem that higher flow aggregation would allow queuing delay to be reduced proportionate to  $1/\sqrt{n}$  without losing utilisation. However, *the dilemma in Figure 3 still applies, so loss probability would increase*. Therefore, once a good balance between queuing delay and loss has been determined it will be best to stick with that for any level of aggregation. Then, on a highly aggregated link, in comparison to an equivalent level of normalised load on a smaller link with fewer flows, queue variation will be smaller but it will have to sit behind a larger standing queue. Nonetheless, this is preferable to changing the AQM slope to trade more loss for less queuing.

This should help to explain why Vu-Brugier *et al* [VBSLS07] found that they could not reduce the size of a buffer to  $\text{BDP}/\sqrt{n}$  without triggering high volumes of user complaints due to excessive loss levels.

If there are too many flows for the capacity, no amount of AQM configuration will help. Capacity needs to be appropriately upgraded. The following formula can be used to determine capacity  $X$  given

the expected number of flows  $n$  and expected average base RTT  $D_R$ , where  $d_q^*$  and  $p^*$  are the values of queuing delay and drop at the design point, as determined above.

$$X = \frac{Kns}{(D_R + d_q^*)\sqrt{p^*}}.$$

Alternatively, the formula can be used to determine the number of flows  $n$  that a particular capacity  $X$  can reasonably be expected to support.

For a bottleneck link, the only way to take advantage of the  $1/\sqrt{n}$  reduction in queuing delay due to aggregation without increasing loss is to over-provide capacity, so that  $n/X$  reduces as much as  $(D_R + d_q^*)$  reduces, thus holding  $p^*$  unchanged.

The necessary degree of overprovisioning can be calculated for the worst case where the overprovisioned link remains as the bottleneck. In this case, the design point for queuing delay at low aggregation,  $d_q^*$  is reduced to  $d_q^*/\sqrt{n}$  at high aggregation, then the over-provisioning factor should be:

$$\frac{X'}{X} = \frac{(D_R + d_q^*)}{(D_R + d_q^*/\sqrt{n})}.$$

For example, if average base RTT,  $D_R = 20ms$  a link for  $100\times$  more flows could have the design point for queuing delay reduced from 20 ms to 2 ms while keeping the loss design point constant at 2%. But only if aggregated capacity is over-provided by  $(20 + 20)/(20 + 2) = 1.8$ , i.e.  $180\times$  more capacity for  $100\times$  more flows.

If a core link is over-provisioned such that it is rarely the bottleneck, none of this analysis applies. Its buffer can be sized at  $BDP/\sqrt{n}$  without sacrificing utilisation but accommodating queue variation due to TCP flows bottlenecked elsewhere, and no regard needs to be taken of the risk of loss, which will be vanishingly small.

### 3 Escaping the TCP Dilemma with ECN

Another insight that can be drawn from this analysis is that the dilemma in [Figure 3](#) disappears with ECN. For ECN, the AQM can mark packets without introducing any impairment. There is therefore no downside to clamping down queuing delay for ECN-capable packets. This proves that it is wrong to treat ECN the same as drop.

When ECN was first standardised [[RFB01](#)], it was defined as equivalent to drop. Even though earlier proposals had considered treating ECN differently, it had to be standardised as equivalent otherwise

no consensus could be reached on how to define its meaning.

It is now known that drop probability should be proportional to the square of ECN marking, and the constant of proportionality need not be standardised [[DSBTB15](#)]. Therefore it might now be feasible to reach consensus on a standard meaning for ECN different from but related to drop.

## 4 Conclusions & Further Work

Although Curvy RED seems to be a useful AQM, we are not necessarily recommending it here. We are merely using the concept of curviness to draw insights.

The curviness parameter of Curvy RED can be considered to represent the operator's policy for the tradeoff between delay and loss whenever load exceeds the intended design point. In contrast, PIE and RED embody a hard-coded policy, which dictates that holding down delay is paramount, at the expense of more loss.

Given losses from short interactive flows (e.g. Web) cause considerable delay to session completion, trading less queuing delay for more loss is unlikely to be the optimal policy to reduce delay. Also, as load increases, it may lead real-time applications such as conversational video and VoIP to degrade or fail sooner. Allowing some additional flex in queuing delay with consequently less increase in loss is likely to give more favourable performance for a mix of Internet applications.

This dilemma can be escaped by using ECN instead of loss to signal congestion. Then queuing delay can be capped and the only consequence is higher marking, not higher drop.

The analysis also shows that, once a design point defining an acceptable queuing delay and loss has been defined, the same configuration can be used for the AQM at any link rate, but only for a similar RTT environment. A shallower queuing delay configuration can be used at high aggregation, but only if the higher loss is acceptable, or if the capacity is suitably over-provisioned. Formulae for all these configuration trade-offs have been provided.

We intend to conduct experiments to give advice on a compromise level of curviness that best protects a range of delay-sensitive and loss-sensitive applications during high load. It will also be necessary to verify the theory for all values of curviness, and for other AQMs without their default parameter settings.

Further research is needed to understand how best to average the queue length, and how best to configure the averaging parameter.

## 5 Acknowledgements

Thanks to Koen De Schepper of Bell Labs for suggesting to use the harmonic mean of the base RTTs, rather than assuming equal RTT flows. Thank you to the following for their review comments: Wolfram Lautenschläger and the participants in the RITE project, especially, Gorry Fairhurst, Michael Welzl, Anna Brunström, David Hayes, Nicolas Kuhn and Naeem Khademi.

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## Document history

| Version   | Date        | Author      | Details of change  |
|-----------|-------------|-------------|--|
| Draft 00A | 19 May 2015 | Bob Briscoe | First Draft  |
| Draft 00B | 23 May 2015 | Bob Briscoe | Explained normalised load  |
| Draft 00C | 23 May 2015 | Bob Briscoe | Changed notation & added Aggregation section   |
| Draft 00D | 08 Jun 2015 | Bob Briscoe | Corrected explanation about packet size, plus minor corrections  |
| Draft 00E | 10 Jun 2015 | Bob Briscoe | Cited DCttH.   |
| Issue 01  | 25 Jul 2015 | Bob Briscoe | Generalised from equal flows; Included Base RTT in Normalised Load; Changed $s$ from packet size to MSS. Justified standing queue due to flow aggregation. |
| Issue 01A | 26 Jul 2015 | Bob Briscoe | Editorial Corrections.   |
| Issue 01B | 29 Jul 2015 | Bob Briscoe | Clarified applicability and more comprehensive advice on scaling AQM config.   |